J. Fluid Mech. (1977), *vol.* 80, *part* 4, *pp.* 795–797
Printed in Great Britain

Effect of external disturbances on the spreading rate of a plane turbulent jet

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(Received 3 December 1976)

The nonlinear growth of turbulent jets documented by Kotsovinos (1976) is tentatively attributed to draughts in the laboratory, caused principally by the jet itself.

1. Introduction

Kotsovinos (1976) shows that the apparently discrepant data for the jet spreading rate obtained by different investigators collapse rather accurately on a plot of jet width against downstream distance. The spreading rate, which would be constant in a self-preserving flow, increases with downstream distance: Kotsovinos tentatively suggests an asymptotic spreading rate of 0·14, but the data are not incompatible with an indefinite increase of spreading rate.

2. Analysis

The most obvious explanation, three-dimensional effects, is apparently demolished by the rather large range of slot aspect ratios used by the different workers whose data collapse together. We should expect – using Kotsovinos' notation – that

$$\frac{b}{D} = f\left(\frac{x}{D}, \frac{W}{D}\right) \tag{1}$$

but the results seem to be independent of W/D. Another possibility is the effect of turbulence in the exit flow, which dies out rather more slowly than might be expected because the component of momentum normal to the plane of the jet is conserved (so that, for instance, the response to very-low-frequency fluctuations would be a bodily flapping of the jet). However the growth rate is initially constant, though larger than for a non-turbulent exit flow, and should decrease to the value for a non-turbulent exit flow once the width of the jet becomes large compared with the wavelength of the exit fluctuations, so that the latter are dissipated by interaction with the natural turbulence. Much the same would be true of other kinds of initial disturbance. Kotsovinos' data analysis, on the other hand, shows an *increase* of growth rate with downstream distance.

Kotsovinos' suggestion that relaminarization of the jet may occur at large downstream distances is not likely to be correct, because the Reynolds number based on the jet width b and centre-line velocity U_m increases as $b^{\frac{1}{2}}$ if the velocity-profile shape remains constant (conservation of axial momentum requiring $U_m^2\, b$ = constant). The root-mean-square turbulent fluctuations are likely to be at least roughly proportional to U_m and therefore vary as $b^{-\frac{1}{2}}$. This, like the last result, follows from conservation of momentum independently of any assumption of self-preservation (which leads to the conventional $b \propto x - x_0$, $U_m \propto (x-x_0)^{-\frac{1}{2}}$) and should not be greatly affected by minor changes in velocity-profile shape. Since U_m and the turbulence level in the jet decrease with increasing distance downstream, they must eventually fall to the same order of magnitude as the fluctuating draughts in the room and this 'free-stream turbulence' will as usual increase the growth rate, by an amount that increases with increasing downstream distance. That is, draughts in the room change the growth rate in the same sense as the data documented by Kotsovinos.

In a well-conducted experiment these draughts will be caused largely, and inevitably, by the continuous recirculation of fluid entrained into the jet. Steady draughts could bend the jet centre-line but have little other effect. The r.m.s. draught velocity in a 'fully stirred' situation will be proportional to the jet exit velocity U_0 with some dependence on the ratio of jet exit area to (room volume)$^{\frac{2}{3}}$. The latter ratio will vary from experiment to experiment but probably not by many orders of magnitude. Very crudely, then, we can represent jet-induced room draughts by an r.m.s. velocity fluctuation $c_1 U_0$, where c_1 is nominally a constant. If the wavelength of the fluctuating draughts is sufficiently long for the jet to be translated sideways (at r.m.s. velocity $c_1 U_0$) without large changes in its turbulence structure the increase in the angle of spread at given x will be proportional to $c_1 U_0/U_m$: if we were considering the diffusion of isolated passive particles rather than a fluid jet the constant of proportionality would be unity. Therefore we write

$$\frac{db}{dx} = \left(\frac{db}{dx}\right)_0 + c_2 \frac{U_0}{U_m}, \tag{2}$$

where c_2 is expected to be of the same order as c_1. To a first approximation (self-preserving flow) $U_m/U_0 = 2{\cdot}5(D/x)^{\frac{1}{2}}$, where D is the slot thickness, so

$$\frac{b}{D} = \left(\frac{db}{dx}\right)_0 \frac{x}{D} + \frac{2c_2}{7{\cdot}5}\left(\frac{x}{D}\right)^{\frac{3}{2}}. \tag{3}$$

A fit to figure 5 of Kotsovinos' paper ($x/D < 200$) suggests $c_2 \simeq 5 \times 10^{-3}$, taking $(db/dx)_0 = 0{\cdot}0913$ following Kotsovinos. The single set of data points in figure 6, for x/D up to 2400, is fitted quite well by (3) with $c_2 \simeq 4 \times 10^{-3}$, again with

$$(db/dx)_0 = 0{\cdot}0913.$$

If $c_1 \approx c_2$ this implies that the background turbulence level is roughly $\frac{1}{2}$ % of the exit velocity. The near-universality of c_2 supports the present hypothesis, suggesting that differences in geometry between one laboratory jet combination and another have little effect on the background turbulence. The result also suggests that natural draughts in the room, which would not scale with U_0, did not have a significant effect in the experiments discussed by Kotsovinos. As pointed out by a referee, an equation analogous to (3), but with a term in x^2 rather than $x^{\frac{3}{2}}$, should apply to round jets: however there appear to be no experiments at really large x/D in that case.

3. Conclusion

A tolerably good fit to the jet-spreading data reviewed by Kotsovinos is obtained by assuming an ambient turbulence level of a few thousandths of the jet exit velocity, and it is suggested that ambient turbulence, due principally to the recirculating flow of the jet itself and therefore having an r.m.s. level related to the jet exit velocity, is the main cause of the observed nonlinear spreading.

I am grateful to Mr D. H. Wood, and to the referees, for helpful comments.

REFERENCE

KOTSOVINOS, N. E. 1976 *J. Fluid Mech.* **77**, 305.